Applied Mathematics

(Statics)

Concepts Sheet

3rd Secondary

Unit One : Friction

The smooth surfaces:
the friction force is completely vanished and the coefficient of friction = zero.
The rough surfaces:
the friction force is appeared, and the coefficient of friction equal a real positive number.

Reaction:

➡ **In case the smooth planes** the reaction is to be perpendicular on the surface of the common tangent of the touching bodies.
➡ **In case of rough planes,** the direction of the reaction is unspecified, and it is depended on the natural of the touching surfaces and the other forces which acts on the body.

The static friction force appears when the two surfaces are sliding or about to move and its direction is in opposite direction to the direction of the body tends to move, and it is given by the inequality
$0 < F \leq \mu_s N$ where μ_s is the static coefficient friction
The limiting static friction force: when the force of friction become the limiting static friction (F_s) the body is about to move (without moving) and the friction is limiting and we symbolize it by (F_s) and $F_s = \mu_s N$

Kinetic friction force: If a body moves on a rough plane, then it is subject to a kinetic friction force its direction is opposite to its motion and the value of it is given by the relation:
$F_k = \mu_k N$ where μ_k is the kinetic coefficient friction.

Notes on coefficients of static and kinetic friction:

➡ μ_s , μ_k depend on the nature of the body and the surface and not depended on the areas of the contiguous surfaces of the mass of the moving body.
➡ Coefficient of static friction (μ_s)> Coefficient of kinetic friction (μ_k)

The resultant reaction: the resultant reaction (R`) is the resultant of the normal reaction $\vec{N}$ and the friction force $\overrightarrow{F_s}$

Angle of friction: the angle which included between the normal reaction and the resultant reaction when the friction is limiting.

The relation between coefficient of friction and angle of friction: when the friction is limiting, then the coefficient of friction equals the tangent of the angle of friction

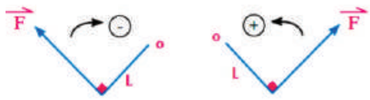
The relation between the measure of angle of friction and the measure of the angle of inclination of the plane to the horizon:
If a body is placed on a rough inclined plane and the body is about to slide under the effect of its weight only , then the measure of the angle of inclination of the plane to the horizontal equals to the measure of the angle of friction.

Unit Two : Moments

1 ***The moment of a force about a point:*** the moment of the force $\vec{F}$ acting on a body about a Point (O) is known as the ability of the force $\vec{F}$ to produce a rotation to the body about point (O) and the moment of the force $\vec{F}$ is calculated by the relation $\overrightarrow{M_o} = \vec{r} \times \vec{F}$ where $\vec{r}$ is the position vector of a point on the line of action of the force about point (O) and the direction of the moment is normal to the plane containing each of $\vec{F}$ and $\vec{r}$.

2 ***The norm of the moment of a force about a point:*** if F represents the norm of the force $\vec{F}$ and L represents the length of the perpendicular segment from point O to the line of action of the force, then the norm of the moment of $\vec{F}$ about point O is calculated by the relation: $\| \overrightarrow{M_o} \| = FL$.

3 ***Algebraic measures of the moment of a force about a point:*** if the force works to rotate the body about point O in the clockwise direction, then the algebraic measure of the moment vector is negative and if the force works to rotate the body about point O in anticlockwise direction, then the algebraic measure of the moment vector is positive.



4 The length of the perpendicular segment from point O to the line of action of the force $\vec{F}$ is L where $L = \frac{\| \overrightarrow{M_o} \|}{\| \vec{F} \|}$

5 If the moment of the force about a point vanishes, then the line of action of this force passes through this point.

6 ***The principle of moments (Varignon's theorem)*** the moment of the force $\vec{F}$ about a point is equal to the sum of the moments of the components of this force about the same point.

7 ***Theorem:*** The sum of the moments of a system of forces acting at a point about any point in space is equal to the moment of the resultant of these forces about the same point.

8 If the sum of the moments of a system of forces about point A = the sum of the moments of these forces about point B, then the line of action of the resultant is parallel to $\overleftrightarrow{AB}$.

9 If the sum of the moments of a system of forces about point A = - the sum of the moments of these forces about point B, then the line of action of the resultant bisects $\overline{AB}$.

10 ***The moment of a force about a point in space***

$$\overrightarrow{M_o} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$ Where:

$\vec{r}$ is the position vector of a point on the line of action of the force with respect to point O.

11 ***The components of the moment of a force in the direction of axes*** if $\vec{F} = (F_x , F_y , F_z)$ is a force acting at a point whose position vector about the origin point O is $\vec{r} = (x, y, z)$ then:

$(y F_z - z F_y)$ the component of the moment of $\vec{F}$ in the direction of the X-axis

$(z F_x - x F_z)$ the component of the moment of $\vec{F}$ in the direction of the Y-axis

$(x\,F_y - y\,F_x)$ the component of the moment of $\vec{F}$ in the direction of the Z-axis

Unit Three : Parallel coplanar forces

1 The resultant of two parallel forces having the same direction (like forces):

$\overrightarrow{F_1} = F_1\,\vec{C}$, $\overrightarrow{F_2} = F_2\,\vec{C}$ act at A and B then:

the resultant $\vec{R} = \overrightarrow{F_1} + \overrightarrow{F_2}$ and acts at point $C \in \overline{AB}$

such that: $\dfrac{AC}{CB} = \dfrac{F_2}{F_1}$

2 The resultant of two parallel forces having opposite directions (unlike forces):

$\overrightarrow{F_1} = F_1\,\vec{C}$, $\overrightarrow{F_2} = -F_2\,\vec{C}$ where $(F_1 > F_2)$ act at A and B then:

$\vec{R} = \overrightarrow{F_1} + \overrightarrow{F_2} = (F_1 - F_2)\,\vec{C}$ and acts at point $C \in \overrightarrow{B\,A}$

such that: $\dfrac{AC}{CB} = \dfrac{F_2}{F_1}$

3 The moments of the parallel coplanar forces:

Theorem (the sum on the moments of any finite number of parallel coplanar force about any point in its plane is equal to the moment of the resultant of these forces about the same point).

4 The resultant of a system of parallel forces:

If the forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ ....., $\overrightarrow{F_n}$ are parallel and acting at points $A_1, A_2, \dots A_n$ then their resultant is $\vec{R}$ where $\vec{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \dots + \overrightarrow{F_n}$ and acts at point C where:

$$A_1C = \frac{||\text{ sum of moments of forces about } A_1||}{||\vec{R}\,||}$$

5 The equilibrium of a system of the parallel coplanar forces:

Rule : If a rigid body is in equilibrium under the action of system of parallel coplanar forces, then:

1- The sum of the algebraic measures of these forces (with respect to a parallel unit vector) is equal to zero (the resultant = zero).

2- The sum of the algebraic measures of the moments of these forces about any point in its plane is equal to zero.

Unit Four : General Equilibrium

If a rigid body is in equilibrium under the action of two forces, then the point of action of any of the two forces can be traveled into another point on its line of action without acting at the equilibrium of the body.

If three coplanar forces meeting at a point, are in equilibrium and a triangle whose sides are parallel to the lines of action of these forces is drawn, then the triangle sides lengths are proportional to the magnitudes of the corresponding forces.

If a body under the action of three coplanar forces but not parallel is in equilibrium, then the lines of action of these forces meet at one point.

The conditions of the equilibrium of a body under the action of a system of coplanar forces meeting at a point:

- The algebraic sum of the components of the forces in the direction of $\overrightarrow{OX}$ = zero
- The algebraic sum of the components of the forces in the direction of $\overrightarrow{OY}$ = zero

The vanishing of the moment of a set of forces about any point:
the moments of rotation acting on a body in the clockwise direction are in equilibrium to the moments of rotation in the anticlockwise direction so that the body is in an equilibrium state.

The sufficient and necessary condition for a set of coplanar forces to be in equilibrium:
For a set of coplanar forces to be in equilibrium, it is necessary and sufficient to satisfy the next conditions:

- The sum of the algebraic components of the forces in the two orthogonal directions lying in their plane vanishes.
- The sum of the algebraic measures of the moment of these forces about one point in their plane vanishes.
- These conditions can be expressed mathematically as follows: $x = 0 , y = 0 , M = 0$

Unit Five : Couples

Definition of couple: it is a system of forces formed from two forces of equal magnitudes and
opposite directions and acting in different lines of action.

The moment of a couple: it is known as the sum of the moments of two forces of the couple about a point in space. Its magnitude is equal to the product of the magnitude of one of the two forces by the distance between them.

Theorem: the moment of a couple is a constant vector independent of the point about which we take the moment of the two forces.

Equilibrium of two couples: the two couples are said to be in equilibrium if the sum of their two moments is the zero vector.

Equilibrium of a body under the action of several couples if several coplanar couples which the direction of their moments are $\overrightarrow{M_1}, \overrightarrow{M_2}, \ldots, \overrightarrow{M_n}$ act a rigid body, then the condition of equilibrium of the body under the action of these couples is $\overrightarrow{M_1} + \overrightarrow{M_2} + \ldots + \overrightarrow{M_n} = \vec{0}$

Equivalent couples: two coplanar couples in the same plane are equivalent if the two algebraic measures of the two moments of their vectors are equal.

The system of the coplanar forces equivalent to a couple:
A system of coplanar forces $\overrightarrow{F_1}, \overrightarrow{F_2}$ ....., $\overrightarrow{F_n}$ is said to be equivalent to a couple if the following two conditions are satisfied together:
1 - The resultant of forces is equal to the zero vector ($\overrightarrow{F_1} + \overrightarrow{F_2} + \ldots\ldots + \overrightarrow{F_n} = \vec{0}$)
2 - The sum of moments of the forces about any point in space does not vanish.

Rule 1: If three coplanar forces act on a rigid body and are completely represented by the sides of a triangle, taken the same way round, then this system is in equivalent to couple the magnitude of its moment is equal to twice the area of the triangle by the magnitude of the force representing the unit length.

Generalization: if a system of coplanar forces acts on a rigid body and are completely represented by the sides of a closed polygon taken the same way round, then this system is equivalent to a couple the magnitude of its moment is equal to the product of twice the surface area of the polygon by the magnitude of the force representing the unit length.

Rule 2: If the sum of the algebraic measures of the moments of a system of coplanar forces with respect to three points in its plane and are not lying in the same straight line is equal to a constant magnitude unequal to zero, then the system is equivalent to a couple the algebraic measure of its moment is equal to this constant magnitude.

Resultant couple: The sum of two coplanar couples $\overrightarrow{M_1}, \overrightarrow{M_2}$ is known as the couple whose moment is equal to the sum of the two moments of those two couples $\vec{M} = \overrightarrow{M_1} + \overrightarrow{M_2}$ and the sum of the two coplanar couples is called the resultant couple (the system is equivalent to a couple):

Unit Six : Center of Gravity

1) **The center of gravity of a rigid body** is a constant point in the body through which the Line of action of the resultant of the weights of the particles from which the body is made up Of whatever the position of the body changes with respect to the ground.

2) **Notes about the center of gravity:**
* The center of gravity of the rigid body changes by the change in its shape due to the change of the dimensions of the particles which the body is made up of.
* The body of a uniform density is the body which the mass of the unit length, areas or volumes taken from any part of it is constant.

3) **The position vector of the center of gravity of a rigid body about the origin point:**

If $m_1, m_2, m_3, \ldots\ldots. m_n$ are the masses of the particles which the rigid body is made up of and, $\overrightarrow{r_1}, \overrightarrow{r_2}, \overrightarrow{r_3}, \ldots\ldots., \overrightarrow{r_n}$ are the position vectors of these particles about the origin point, then the position vector $\overrightarrow{r_G}$ of the center of gravity of the rigid body about the origin point is identified by the relation:

$$\overrightarrow{r_G} = \frac{m_1\overrightarrow{r_1} + m_2\overrightarrow{r_2} + m_3\overrightarrow{r_3} + \ldots\ldots + m_n\overrightarrow{r_n}}{m_1 + m_2 + m_3 + \ldots\ldots + m_n}$$

It is expressed in terms of the components of the center of gravity in the orthogonal Cartesian coordinate system as follows:

$$X_G = \frac{m_1X_1 + m_2X_2 + m_3X_3 + \ldots\ldots + m_nX_n}{m_1 + m_2 + m_3 + \cdots\ldots + m_n} \quad ,$$

$$Y_G = \frac{m_1Y_1 + m_2Y_2 + m_3Y_3 + \ldots\ldots + m_nY_n}{m_1 + m_2 + m_3 + \ldots\ldots + m_n}$$

4) **The free suspension of the rigid body:**

The center of gravity of the rigid body suspended freely lies on the vertical straight line passing through the suspension point.

5) **The center of gravity of a uniform fine rod:**

The center of gravity of a fine rod of a uniform density lies at its midpoint.

6) **The center of gravity of a uniform fine lamina in the form of a parallelogram:**

The center of gravity of a uniform lamina bounded by a parallelogram lies at its geometrical

center C (intersection point of the diagonals of the parallelogram).

7) **The center of gravity of a uniform fine lamina in the form of a triangle:**
The center of gravity of a uniform lamina bounded by a triangle lies at the intersection point of the medians of the triangle.

8) **The negative mass method:**
Considering the mass of the origin body is (m) and the part cut off (considering its mass is negative) is $(-m_1)$, then the mass of the remaining part is $(m - m_1)$ thus $\overrightarrow{r_2}$ is given by the relation:

$$\overrightarrow{r_2} = \frac{m\overrightarrow{r_G} - m_1\overrightarrow{r_1}}{m - m_1}$$

The previous vector relation can be written in terms of the components in the direction of the orthogonal coordinates: $\overrightarrow{OX}$, $\overrightarrow{OY}$, we obtain the follows:

$$X_2 = \frac{mX_G - m_1X_1}{m - m_1} \quad , \quad Y_2 = \frac{mY_G - m_1Y_1}{m - m_1}$$

9) **The symmetry of a fine geometrical lamina of a uniform density:**
If a geometrical axis of symmetry for a fine lamina of a uniform density exists, then its center
of gravity lies on the line of this axis.

10) **The symmetry of a geometrical solid of a uniform density:**
If a geometrical plane of symmetry for a solid of a uniform density is existed then its center of gravity lies in this plane.

11) **Some special cases of the center of gravity:**
⟹The center of gravity of a wire of a uniform density in the form of a circle lies in the center of the circle.
⟹The center of gravity of a lamina of a uniform density in the form of a circle lies in the center of the circle.
⟹The center of gravity of a spherical crust (cortex) of a uniform density lies in the center of the sphere.
⟹The center of gravity of a solid sphere of a uniform density lies in the center of the sphere.
⟹The center of gravity of a solid of a uniform density in the form of a cuboid lies in its geometrical center.
⟹The center of gravity of a right circular cylinderic crust of a uniform density lies at the midpoint of the line segment joins the center of its base.
⟹The center of gravity of a right solid circular cylinder of a a uniform density lies at the midpoint of the line segment joins the center of its base .
⟹The center of gravity of a uniform right prism lies at the midpoint of the axis parallel to its lateral edges and passing through the two centers of gravity of its two bases considering them as two fine laminas of uniform densities.

Mathématiques Appliquées

(Statiques)

Résumé de concepts

3rd Secondary